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(Residential Autonomous College under University of Calcutta)

M.A./M.Sc. THIRD SEMESTER EXAMINATION, JANUARY 2015

SECOND YEAR

Date : 16/01/2015 **MATHEMATICS**

Time : 11 am – 1 pm Paper : XIII [Graph Theory & Combinatorics] Full Marks : 50

Answer <u>any two</u> questions from <u>Q.No 1 - 3</u> : [2×10]

1. a) How many numbers between 400 and 1000 can be formed with the digits 2,3,4,5,6 and 0, if repetition of digits is not allowed? [2]

   b) Five gentlemen A, B, C, D and E attend a party, where before joining the party, they leave their overcoats in a check room. After the party the overcoats get mixed up and are returned to the gentlemen in a random manner. Find the number of ways in which none receives his own overcoat. [4]

   c) How many solutions are there of the equation $x+y+z+w=20$ in which x, y, z, w are integers such that x is positive, y is at least 7, z, $w \geq 0$? [4]

2. a) State Pigeonhole Principle.
   Hence, for a set A of some fixed 10-element subset of {1, 2, 3, ..., 50}, show that A posseses two different 5-element subsets, the sums of whose elements are equal. [2+3]

   b) How many bit strings of 0, 1 of length 10 contain at least 5 consecutive 0s or 5 consecutive 1s? [3]

   c) Find the generating function of the following sequence :
   3, –3, 3, –3, 3, –3, ... [2]

3. a) In the Tower of Hanoi Puzzle, suppose our goal is to transfer all n disks from peg 1 to peg 3, but we cannot move a disk directly between peg 1 and peg 3, each move of a disk must be a move involving peg 2. Find a recurrence relation for the number of moves required to solve the puzzle for n disks with initial condition. [5]

   b) Solve the following recurrence relation : $y_{n+2}-y_{n+1}-2y_n=n^2$. [5]

Answer <u>any three</u> questions from <u>Q.No 4 - 8</u> : [3×10]

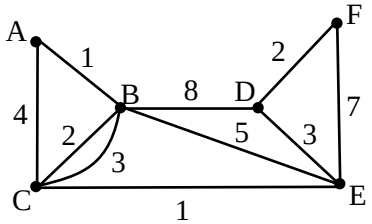
4. Explain the following terms (for graph) with example (**<u>any four</u>**): [4×2·5]
   a) 1-isomorphism
   b) Geometric dual of pseudograph
   c) Diameter and eccentricity
   d) Cut and cut-set
   e) Fusion

5. a) Does there exists a simple graph corresponding to the following degree sequences (give reasons)? [2]
   (i) (2, 2, 4, 6) (ii) (1, 3, 3, 4, 5, 6, 6)

   b) Prove that a simple graph with n vertices and k components can have at most $(n-k)(n-k+1)/2$ edges. [3]

   c) Prove that a connected graph G is an Euler graph if and only if the vertices of G are of even degrees. [5]

6. a) Applying Dijkstra's algorithm find the shortest path and the length of the shortest path from A to F in the following graph: [5]



b) For a connected plane graph with n-vertices, e-edges and f-faces, show that $n - e + f = 2$. [5]

7. a) Write down an algorithm to find a minimal spanning tree for a connected weighted graph. [3]

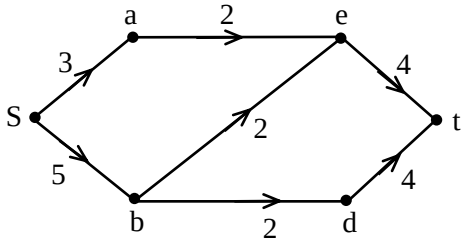
b) Draw the binary tree with node lebels a, b, c, d, e, f and g for which the inorder and post order traversals result in the following sequences : [3]

Inorder	a	f	b	c	d	g	e
Postorder	a	f	c	g	e	d	b

c) If one row of the incedence matrix of a graph has all entries 0(zero), what information can you deduce about the graph? [2]

d) What is k-critical graph? [2]

8. a) Using Ford-Fulkerson algorithm find the maximum flow for the following network. Also find the cut with capacity equal to this maximum flow. [5+1]



b) Prove that a covering g of a graph is minimal if and only if g contains no path of length three or more. [3]

c) What is chromatic index? [1]

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